

MQDSS

Ming-Shing Chen¹, **Andreas Hülsing**², Joost Rijneveld³, Simona Samardjiska³,
and Peter Schwabe³

- ¹ National Taiwan University / Academia Sinica, Taipei, Taiwan
- ² Technische Universiteit Eindhoven, Eindhoven, The Netherlands
- ³ Radboud University, Nijmegen, The Netherlands

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Second NIST PQC Standardization Conference

In a nutshell..

- ▶ \mathcal{MQ} -based 5-pass identification scheme
 - ▶ Fiat-Shamir transform
- ▶ Loose reduction from (only!) \mathcal{MQ} problem
 - ▶ Security proof, instead of typical 'break and tweak' in \mathcal{MQ} cryptography
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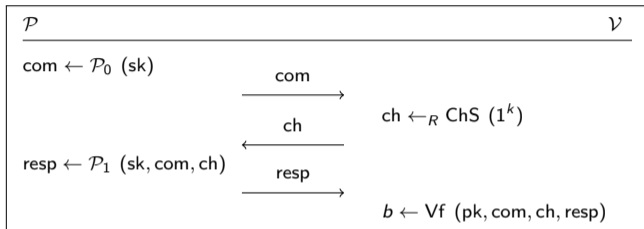
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- ▶ **New in Second Round submission**
 - ▶ Reduction of number of rounds
 - ▶ Added randomness in commitments
 - ▶ more precise analysis of best attacks against \mathcal{MQ}

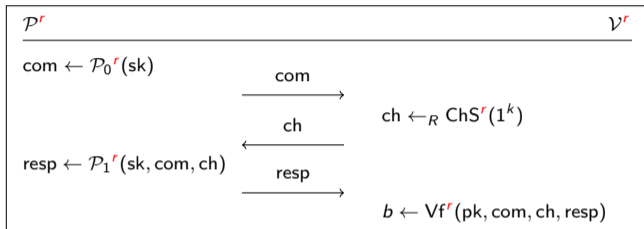
Fiat-Shamir transform

IDS



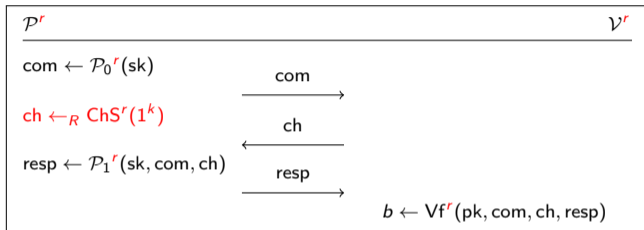
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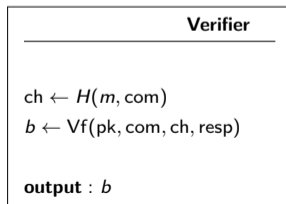
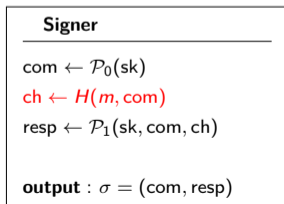


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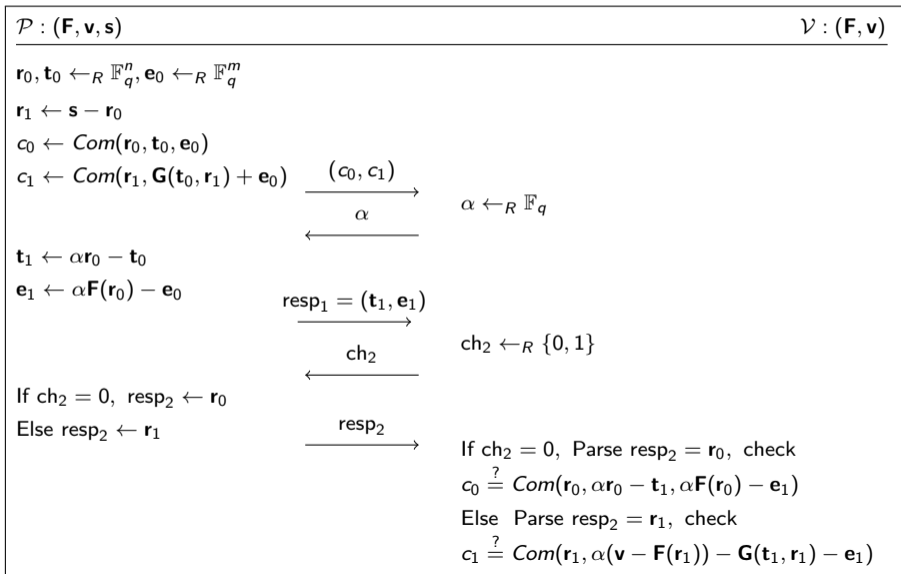
IDS



FS signature



Sakumoto-Shirai-Hiwatari 5-pass IDS [SSH11]



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- ▶ Generate keys
 - ▶ Sample seed $\mathcal{S}_F \in \{0, 1\}^k$, $\mathbf{sk} \in \mathbb{F}_q^n \Rightarrow (\mathcal{S}_F, \mathbf{sk})$
 - ▶ Expand \mathcal{S}_F to \mathbf{F} , compute $\mathbf{pk} = \mathbf{F}(\mathbf{sk}) \Rightarrow (\mathcal{S}_F, \mathbf{pk})$

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 - ▶ Perform r parallel rounds of transformed IDS
 - ▶ Sample r vectors \mathbf{r} , \mathbf{t} and \mathbf{e}
 - ▶ $2r$ commitments, some multiplications in \mathbb{F}_q
 - ▶ $2r$ \mathcal{MQ} evaluations

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- ▶ Parameters: n , m , \mathbb{F}_q , Com, hash functions, PRGs

New in Round 2: Parameter Sets

	Sec. cat.	q	n (= m)	r	pk (bytes)	sk (bytes)	Signature (bytes)
MQDSS-31-48 (Round 1)	1-2	31	48	135 269	46 62	16 32	20854 32882
MQDSS-31-64 (Round 1)	3-4	31	64	202 403	64 88	24 48	43728 67800

Table: Round 1 parameters in black, Round 2 parameters in red.

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- ▶ $q, n = m$ chosen using best attacks on \mathcal{MQ}
 - ▶ q additionally chosen for fast arithmetic
- ▶ r chosen such that $2^{-(r \log \frac{2q}{q+1})} < 2^{-k}$
 - ▶ **mistake in calculation in Round 1, chose k too large**

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- ▶ **Round 2:**
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 - ▶ Still needs randomness - $2\times$ commitment length [Lei18]
 - ▶ \Rightarrow adds approx 4KB (10KB) to signature for MQDSS-31-48 (MQDSS-31-64)

Round 2 performance

► Reference implementation

	keygen	signing	verification
MQDSS-31-48	1 192 984	26 630 590	19 840 136
Round 1	1 206 730	52 466 398	38 686 506
MQDSS-31-64	2 767 384	85 268 712	62 306 098
Round 1	2 806 750	169 298 364	123 239 874

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► AVX2 implementation

	keygen	signing	verification
MQDSS-31-48	1 074 644	3 816 106	2 551 270
MQDSS-31-64	2 491 050	9 047 148	6 132 948

Round 2: More precise analysis of hardness of \mathcal{MQ}

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- ▶ Analyze both classically and using Grover
 - ▶ Classical gates, quantum gates, circuit depth
 - ▶ minor changes in **Round 2** - more precise analysis
 - ▶ no influence to security of parameter sets

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- ▶ Can be mitigated by $\approx 1.4 \times$ (number of rounds)
- ▶ **Proof still valid!**
 - ▶ Attack is result of not taking into account non-tightness of proof for choosing parameters
- ▶ **New parameters after attack (estimate):**

	Sec. cat.	q	n	r	pk	sk	Signature
MQDSS-31-48 (new) Round 1	1-2	31	48	184 269	46B 62B	16B 32B	28400B 32882B
MQDSS-31-64 (new) Round 1	3-4	31	64	277 403	64B 88B	24B 48B	59928B 67800B

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Thank you for your attention!

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